PERGAMON

Computers and Electrical Engineering 28 (2002) 241–253

Computers and Electrical Engineering

www.elsevier.com/locate/compeleceng

Adaptive control of dynamic mobile robots with nonholonomic constraints

Farzad Pourboghrat *, Mattias P. Karlsson

Department of Electrical and Computer Engineering, Southern Illinois University, Carbondale, IL 62901-6603, USA

Received 16 November 1999; accepted 22 August 2000

Abstract

This paper presents adaptive control rules, at the dynamics level, for the nonholonomic mobile robots with unknown dynamic parameters. Adaptive controls are derived for mobile robots, using backstepping technique, for tracking of a reference trajectory and stabilization to a fixed posture. For the tracking problem, the controller guarantees the asymptotic convergence of the tracking error to zero. For stabilization, the problem is converted to an equivalent tracking problem, using a time varying error feedback, before the tracking control is applied. The designed controller ensures the asymptotic zeroing of the stabilization error. The proposed control laws include a velocity/acceleration limiter that prevents the robot's wheels from slipping.

Keywords: Mobile robot; Nonholonomic constraint; Dynamics level motion control; Stabilization and tracking; Adaptive control; Backstepping technique; Asymptotic stability

1. Introduction

Motion control of mobile robots has found considerable attention over the past few years. Most of these reports have focused on the steering or trajectory generation problem at the kinematics level i.e., considering the system velocities as control inputs and ignoring the mechanical system dynamics [1–3]. Very few reports have been published on control design in the presence of uncertainties in the dynamic model [4]. Some preliminary results on control of nonholonomic systems with uncertainties are given in Refs. [4–6].

Two of the most important control problems concerning mobile robots are tracking of a reference trajectory and stabilization to a fixed posture. The tracking problem has received solutions

* Corresponding author. Tel.: +1-618-453-7026.
E-mail address: pour@siu.edu (F. Pourboghrat).

0045-7906/02/$ - see front matter
PII: S0045-7906(00)00053-7

including classical nonlinear control techniques [1,2,7]. The basic idea is to have a reference car that generates a trajectory for the mobile robot to follow. In Refs. [1,2], nonlinear velocity control inputs were defined that made the tracking error go to zero as long as the reference car was moving. In Ref. [7], they used input–output linearization to make a mobile platform follow a desired trajectory.

The problem of stabilization about a fixed posture has been shown to be rather complicated. This is due to violating the Brockett's condition [8], which states that for nonholonomic systems a single equilibrium solution cannot be asymptotically stabilized using continuous static state feedback [9,10]. The Brockett's condition essentially states that for nonholonomic systems an equilibrium solution can be asymptotically stabilized only by either a time varying, a discontinuous, or a dynamic state feedback.

In addressing the above problem, in Ref. [10] a smooth feedback control was presented for the kinematics control problem resulting in a globally marginally stable closed loop system. They also designed a smooth feedback control for a dynamical state-space model resulting in a Lagrange stable closed loop system, as defined in their paper. A two dimensional Lyapunov function was utilized in Ref. [3] to prescribe a set of desired trajectories to navigate a mobile robot to a specified configuration. The desired trajectory was then tracked using sliding mode control, resulting in discontinuous control signals. The mobile robot was shown to be exponentially stable for a class of quadratic Lyapunov functions. In Ref. [9], they formulated a reduced order state equation for a class of nonholonomic systems. Several other researchers have later used this reduced order state equation in their studies. In Ref. [4], the problem of controlling nonholonomic mechanical systems with uncertainties, at the dynamics level, was considered. Using the reduced state equation in Ref. [9], they proposed an adaptive controller for a number of important nonholonomic control problems, including stabilization of general systems to an equilibrium manifold and stabilization of differentially flat and Caplygin systems to an equilibrium point. In Ref. [2], they gave several examples on how the stabilization problem can be solved for a mobile robot at the kinematics level. Their solutions included time-varying control, piecewise continuous control, and time-varying piecewise continuous control. They also showed how a solution to the tracking problem could be extended to work even for the stabilization problem.

Here, we present adaptive control schemes for the tracking problem and for the problem of stabilization to a fixed posture when the dynamic model of the mobile robot contains unknown parameters. Our work is based on, and can be seen as an extension of, the work presented in Refs. [1,2]. Using backstepping technique we derive adaptive control laws that work even when the model of the dynamical system contains uncertainties in the form of unknown constants. The assumption for the uncertainty in robot's parameters, particularly the mass, and hence the inertia, can be justified in real applications such as in automotive manufacturing industry and warehouses, where the robots are to move a variety of parts with different shapes and masses. In these cases, the robot's mass and inertia may vary up to 10% or 20%, justifying an adaptive control approach.

2. Dynamic model of mobile robot

Here, we consider a three-wheeled mobile robot moving on a horizontal plane (Fig. 1). The mobile robot features two differentially driven rear wheels and a castor front wheel. The radius of

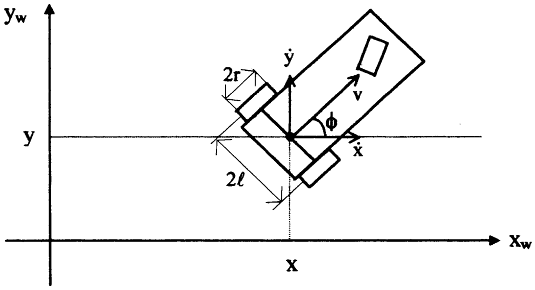

Fig. 1. Mobile robot configuration.

the wheels is denoted r and the length of the rear wheel axis is $2l$. Inputs to the system are two torques T_1 and T_2, provided by two motors attached to the rear wheels.

The dynamic model for the above wheeled-mobile robot is given by Refs. [10,11].

$$\begin{cases} \ddot{x} = \frac{\lambda}{m}\sin\phi + b_1u_1\cos\phi \\ \ddot{y} = -\frac{\lambda}{m}\cos\phi + b_1u_1\sin\phi \\ \ddot{\phi} = b_2u_2 \end{cases} \quad (1)$$

$$\dot{x}\sin\phi - \dot{y}\cos\phi = 0 \quad (2)$$

where $b_1 = 1/(rm)$, $b_2 = l/(rI)$, and that m and I denote the mass and the moment of inertia of the mobile robot, respectively. Also, $u_1 = T_1 + T_2$ and $u_2 = T_1 - T_2$ are the control inputs, and λ is the Lagrange multiplier, given by $\lambda = -m\dot{\phi}(\dot{x}\cos\phi + \dot{y}\sin\phi)$. Here, it is assumed that b_1 and b_2 are unknown constants with known signs. The assumption that the signs of b_1 and b_2 are known is practical since b_1 and b_2 represent combinations of the robot's mass, moment of inertia, wheel radius, and distance between the rear wheels. Eq. (2) is the nonholonomic constraint, coming from the assumption that the wheels do not slip. The triplet vector function $q(t) = [x(t), y(t), \phi(t)]^T$ denotes the trajectory (position and orientation) of the robot with respect to a fixed workspace frame. That is, at any given time, $q = [x, y, \phi]^T$ describes the robot's configuration (posture) at that time. We assume that, at any time, the robot's posture, $q = [x, y, \phi]^T$, as well as its derivative, $\dot{q} = [\dot{x}, \dot{y}, \dot{\phi}]^T$, are available for feedback.

3. Tracking problem definition

The tracking problem consists of making the trajectory q of the mobile robot follow a reference trajectory q_r. The reference trajectory $q_r(t) = [x_r(t), y_r(t), \phi_r(t)]^T$ is generated by a reference vehicle/robot whose equations are

$$\begin{cases} \dot{x}_r = v_r\cos\phi_r \\ \dot{y}_r = v_r\sin\phi \\ \dot{\phi}_r = \omega_r \end{cases} \quad (3)$$

The subscript "r" stands for reference, and v_r and ω_r are the reference translational (linear) velocity and the reference rotational (angular) velocity, respectively. We assume that v_r and ω_r, as well as their derivatives are available and that they all are bounded.

Assumption A_1. For the tracking problem it is assumed that the reference velocities v_r and ω_r do not both go to zero simultaneously. That is, it is assumed that at any time either $\lim_{t\to\infty} v_r(t) \nrightarrow 0$ and/or $\lim_{t\to\infty} \omega_r(t) \nrightarrow 0$.

The tracking problem, under the Assumption A_1, is to find a feedback control law $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = u(q, \dot{q}, q_r, v_r, \omega_r, \dot{v}_r, \dot{\omega}_r)$ such that $\lim_{t\to\infty} \tilde{q}(t) = 0$, where $\tilde{q}(t) = q_r(t) - q(t)$ is defined as the trajectory tracking error. As in Ref. [1], we define the equivalent trajectory tracking error as

$$e = T\tilde{q} \tag{4}$$

where $e = [e_1, e_2, e_3]^T$, and $T = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Note that since T matrix is nonsingular, e is nonzero as long as $\tilde{q} \neq 0$. Assuming that the angles ϕ_r and ϕ are given in the range $[-\pi, \pi]$, we have the equivalent trajectory tracking error $e = 0$ only if $q = q_r$. The purpose of the tracking controller is to force the equivalent trajectory tracking error e to 0. In the sequel we refer to e as the trajectory tracking error.

Using the nonholonomic constraint (2), the derivative of the trajectory tracking error given in Eq. (4) can be written as, [1],

$$\begin{cases} \dot{e}_1 = e_2\omega - v + v_r \cos e_3 \\ \dot{e}_2 = -e_1\omega + v_r \sin e_3 \\ \dot{e}_3 = \omega_r - \omega \end{cases} \tag{5}$$

where v and ω are the translational and rotational velocities of the mobile robot, respectively, and are expressed as

$$\begin{aligned} v &= \dot{x}\cos\phi + \dot{y}\sin\phi \\ \omega &= \dot{\phi} \end{aligned} \tag{6}$$

4. Tracking controller design

Here, the goal is to design a controller to force the tracking error $e = [e_1, e_2, e_3]^T$ to zero. Using backstepping technique, since the actual control variables u_1 and u_2 do not appear in Eq. (5), we consider variables v and ω as virtual controls. Let v_d and ω_d denote the desired virtual controls for the mobile robot. That is, with v_d and ω_d the trajectory tracking error e converges to zero asymptotically. Also let us define $\tilde{v}$ and $\tilde{\omega}$ as virtual control errors. Then, v and ω can be written as

$$\begin{aligned} v &= v_d + \tilde{v} \\ \omega &= \omega_d + \tilde{\omega} \end{aligned} \tag{7}$$

Let us choose the virtual controls v_d and ω_d, as

$$\begin{aligned} v_d(v_r,\omega_r,e_1,e_3) &= v_r\cos e_3 + k_1(v_r,\omega_r)e_1 \\ \omega_d(v_r,\omega_r,e_2,e_3) &= \omega_r + k_2 v_r e_2 + k_3(v_r,\omega_r)\sin e_3 \end{aligned} \tag{8}$$

where k_2 is a positive constant and $k_1(\cdot)$ and $k_3(\cdot)$ are bounded continuous functions with bounded first derivatives, strictly positive on $\Re \times \Re$-$(0,0)$. Observe that our approach from here on is general for any v_d and ω_d (with well defined first derivatives), i.e. any differentiable control law that makes the kinematics model of the mobile robot track a desired trajectory can be used instead of Eq. (8). Eq. (8) is similar to the control law proposed by Ref. [1], but with the advantage, as we are going to prove later, that it can be used to track any reference trajectory as long as Assumption A_1 holds.

Now, consider the following adaptive control scheme:

$$\begin{aligned} u_1 &= \hat{\beta}_1(-c_1\tilde{v} + e_1 + \dot{v}_d) \\ u_2 &= \hat{\beta}_2\left(-c_2\tilde{\omega} + \frac{1}{k_2}\sin e_3 + \dot{\omega}_d\right) \\ \dot{\hat{\beta}}_1 &= -\gamma_1 \mathrm{sign}(b_1)\tilde{v}(-c_1\tilde{v} + e_1 + \dot{v}_d) \\ \dot{\hat{\beta}}_2 &= -\gamma_2 \mathrm{sign}(b_2)\tilde{\omega}\left(-c_2\tilde{\omega} + \frac{1}{k_2}\sin e_3 + \dot{\omega}_d\right) \end{aligned} \tag{9}$$

where c_1, c_2, γ_1, and γ_2 are positive constants and $\hat{\beta}_1$ is an estimate of $\beta_1 = 1/b_1$ and $\hat{\beta}_2$ is an estimate of $\beta_2 = 1/b_2$.

Result 1. *If Assumption A_1 holds, then the adaptive control* scheme (9) *makes the origin $e = 0$ uniformly asymptotically stable.*

Proof. Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}\left(e_1^2 + e_2^2\right) + \frac{1}{k_2}(1 - \cos e_3) \tag{10}$$

where k_2 is a positive constant. Clearly V_1 is positive definite and $V_1 = 0$ only if $e = 0$.

Taking the time derivative of V_1, we obtain

$$\dot{V}_1 = e_1(-v + v_r\cos e_3) + e_2 v_r \sin e_3 + \frac{1}{k_2}\sin e_3(\omega_r - \omega) \tag{11}$$

Furthermore, using Eqs. (7) and (8), we have

$$\dot{V}_1 = -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 - \tilde{v}e_1 - \tilde{\omega}\frac{1}{k_2}\sin e_3 \tag{12}$$

In view of Eqs. (1), (2) and (6), we find the time derivatives of $\tilde{v}$ and $\tilde{\omega}$, as

$$\begin{aligned} \dot{\tilde{v}} &= \dot{v} - \dot{v}_d = \ddot{x}\cos\phi - \dot{x}\sin\phi\dot{\phi} + \ddot{y}\sin\phi + \dot{y}\cos\phi\dot{\phi} - \dot{v}_d = b_1u_1 - \dot{v}_d \\ \dot{\tilde{\omega}} &= \dot{\omega} - \dot{\omega}_d = \ddot{\phi} - \dot{\omega}_d = b_2u_2 - \dot{\omega}_d \end{aligned} \tag{13}$$

Consider the Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}(\tilde{v}^2 + \tilde{\omega}^2) + \frac{|b_1|}{2\gamma_1}\tilde{\beta}_1^2 + \frac{|b_2|}{2\gamma_2}\tilde{\beta}_2^2 \tag{14}$$

where $\tilde{\beta}_1 = \beta_1 - \hat{\beta}_1 = 1/b_1 - \hat{\beta}_1$ and $\tilde{\beta}_2 = \beta_2 - \hat{\beta}_2 = 1/b_2 - \hat{\beta}_2$. Considering Eq. (9) we get:

$$\dot{V}_2 = -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 - c_1\tilde{v}^2 - c_2\tilde{\omega}^2 \leqslant 0 \tag{15}$$

Since V_2 is bounded from below and $\dot{V}_2$ is negative semi-definite, V_2 converges to a finite limit. Also, V_2, as well as, e_1, e_2, e_3, $\tilde{v}$, $\tilde{\omega}$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are all bounded.

Furthermore, using Eqs. (5), (7)–(9) and (13), the second derivative of V_2 can be written as

$$\begin{aligned}\ddot{V}_2 = {} & -2k_1e_1e_2(\omega_r + k_2v_re_2 + k_3\sin e_3 + \tilde{\omega}) + 2k_1e_1(k_1e_1 + \tilde{v}) - \dot{k}_1e_1^2 \\ & + \frac{2k_3}{k_2}\cos e_3 \sin e_3(k_2v_re_2 + k_3\sin e_3 + \tilde{\omega}) - \frac{\dot{k}_3}{k_2}\sin^2 e_3 - 2c_1\tilde{v}(b_1\hat{\beta}_1(-c_1\tilde{v} + e_1 + \dot{v}_d) - \dot{v}_d) \\ & - 2c_2\tilde{\omega}\left(b_2\hat{\beta}_2\left(-c_2\tilde{\omega} + \frac{1}{k_2}\sin e_3 + \dot{\omega}_d\right) - \dot{\omega}_d\right)\end{aligned} \tag{16}$$

which from the properties of k_1, k_2, and k_3, the assumption that v_r and ω_r and their derivatives are bounded, and from the above results, can be shown to be bounded, i.e., $\dot{V}_2$ is uniformly continuous. Since $V_2(t)$ is differentiable and converges to some constant value and that $\ddot{V}_2$ is bounded, by Barbalat's lemma, $\dot{V}_2(t) \to 0$ as $t \to \infty$. This in turn implies that e_1, e_3, $\tilde{v}$, and $\tilde{\omega}$ converge to zero [12,13]. To show that e_2 also goes to zero, note that, using the above results, the first error equation can be written as

$$\dot{e}_1 = e_2\omega_r - k_1e_1 \tag{17}$$

The second derivative of e_1 is

$$\ddot{e}_1 = \dot{\omega}_re_2 + \omega_r(-e_1\omega + v_r\sin e_3) - k_1(e_2\omega_r - k_1e_1) - \dot{k}_1e_1 \tag{18}$$

which can be shown to be bounded by once again using the properties of k_1, the assumptions on v_r and ω_r, and Eqs. (7) and (8). Since e_1 is differentiable and converges to zero and $\ddot{e}_1$ is bounded, by Barbalat's lemma, $\dot{e}_1$, and hence, $e_2\omega_r$ tend to zero. Proceeding in the same manner, the third error equation can be written as

$$\dot{e}_3 = -k_2v_re_2 - k_3\sin e_3 \tag{19}$$

and its second derivative can be shown to be bounded. Since e_3 is differentiable and converges to zero and $\ddot{e}_3$ is bounded, again by Barbalat's lemma, $\dot{e}_3 \to 0$ as $t \to \infty$. Hence, $k_2v_re_2$ and thus v_re_2 tend to zero as $t \to \infty$. Clearly, both v_re_2 and ω_re_2 converge to zero. However, since v_r and ω_r do not both tend to zero (by Assumption A_1), e_2 must converge to zero. That is, e_1, e_2, e_3, $\tilde{v}$, and $\tilde{\omega}$ must all converge to zero. □

In Section 3, we demonstrated that the system is stable if k_2 is a positive constant, and that $k_1(\cdot)$ and $k_3(\cdot)$ are bounded continuous functions with bounded first derivatives and are strictly positive on $\Re \times \Re$-$(0,0)$. To get a better understanding on how the control gains affect the response of the system, we write the equations for the closed loop system when $\tilde{v}$ and $\tilde{\omega}$ are equal to zero as [1]

$$\dot{e} = \begin{pmatrix} -k_1 e_1 + (\omega_r + k_2 v_r e_2 + k_3 \sin e_3) e_2 \\ -(\omega_r + k_2 v_r e_2 + k_3 \sin e_3) e_1 + v_r \sin e_3 \\ -k_2 v_r e_2 - k_3 \sin e_3 \end{pmatrix} \tag{20}$$

By linearizing the differential equation (20) around $e = 0$, we get

$$\dot{e} = Ae \tag{21}$$

where

$$A = \begin{pmatrix} -k_1 & \omega_r & 0 \\ -\omega_r & 0 & v_r \\ 0 & -k_2 v_r & -k_3 \end{pmatrix} \tag{22}$$

To simplify the analysis, we assume that v_r and ω_r are constants. The system's closed loop poles are now equal to the roots of the following characteristic polynomial equation:

$$(s + 2\xi\omega_0)(s^2 + 2\xi\omega_0 s + \omega_0^2) \tag{23}$$

where ξ and ω_0 are positive real numbers. The corresponding control gains are

$$\begin{aligned} k_1 &= 2\xi\omega_0 \\ k_2 &= \frac{\omega_0^2 - \omega_r^2}{v_r^2} \\ k_3 &= 2\xi\omega_0 \end{aligned} \tag{24}$$

With a fixed pole placement strategy (ξ and ω_0 are constant), the control gain k_2 increases without bound when v_r tends to zero. One way to avoid this is by letting the closed loop poles depend on the values of v_r and ω_r. As in Ref. [2], we choose $\omega_0 = (\omega_r^2 + b v_r^2)^{(1/2)}$ with $b > 0$. The control gains then become

$$\begin{aligned} k_1 &= 2\xi(\omega_r^2 + b v_r^2)^{1/2} \\ k_2 &= b \\ k_3 &= 2\xi(\omega_r^2 + b v_r^2)^{1/2} \end{aligned} \tag{25}$$

and the resulting control is now defined for any values of v_r and ω_r.

In the above, it is shown that the proposed algorithm works for any desired velocities, (v_d, ω_d). However, in practice, if the tracking errors initially are large or if the reference trajectory does not have a continuous curvature (e.g., if the reference trajectory is a straight line connected to a circle segment), either or both of the virtual reference velocities in Eq. (8) might become too large for a real robot to attain in practice. Hence, the translational/rotational acceleration might become too large causing the robot to slip [1]. In order to prevent the mobile robot from slipping, in a real application, a simple velocity/acceleration limiter may be implemented [1], as shown in Fig. 2. This limits the virtual reference velocities (v_d, ω_d) by constants (v_{max}, ω_{max}) and the virtual reference accelerations (a, α) by constants (a_{max}, α_{max}), where $a = \dot{v}_d$ and $\alpha = \dot{\omega}_d$ are the virtual reference accelerations. In practice, these parameters must be determined experimentally as the largest values with which the mobile robot never slips.

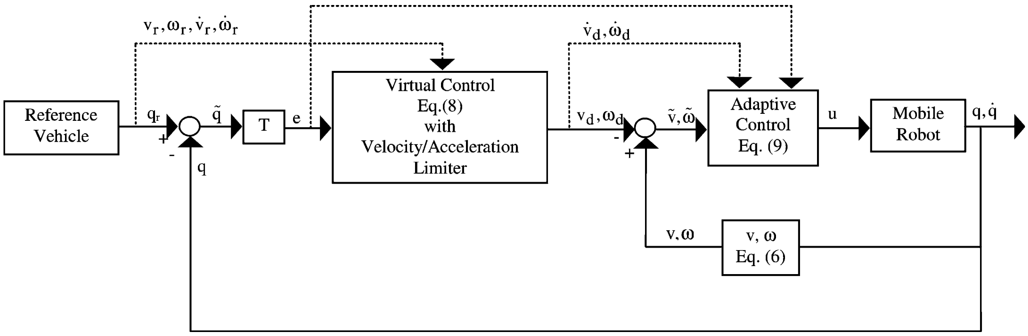

Fig. 2. Tracking control structure.

An important advantage of adding the limiter is that it lowers the control gains indirectly only when the tracking errors are large, i.e., when too high a gain could cause the robot to slip, while for small tracking errors it does not affect the performance at all. Thus, by using the limiter one can have higher control gains for small tracking errors to allow for better tracking, while letting the limiter to "scale down" the gains, indirectly, for large tracking errors, to prevent the robot from slipping.

5. Simulation results for tracking control problem

Here, the results of computer simulation, using MATLAB/SIMULINK, are presented for a mobile robot with the proposed tracking control and with the velocity/acceleration limiter. The computer simulations for the above controller without the limiter, although not shown here, produce similar results, but with somewhat different transient characteristics. All simulations have the common parameters of $c_1 = c_2 = 100$, $\gamma_1 = \gamma_2 = 10$ and $b = 250$. Also selected are, the damping factor $\xi = 1$, $v_{\max} = 1.5$ m/s, $\omega_{\max} = 3$ rad/s, $a_{\max} = 5$ m/s^2 and $\alpha_{\max} = 25$ rad/s^2. Moreover, the robot's dynamic parameters are chosen as $b_1 = b_2 = 0.5$, which are assumed to be unknown to the controller, but with known signs.

Simulation results for the case where the reference trajectory is a straight line are shown in Figs. 3 and 4 for $t \in [0, 10]$. The reference trajectory is given by $x_r(t) = 0.5t$, $y_r(t) = 0.5t$ and $\phi_r(t) = \pi/4$, defining a straight line, starting from $q_r(0) = [x_r(0), y_r(0), \phi_r(0)]^T = [0, 0, \pi/4]^T$. The mobile robot, however, is initially at $q(0) = [x(0), y(0), \phi(0)]^T = [1, 0, 0]^T$, where $\phi = 0$ indicates that the robot is heading toward positive direction of x.

As it can be seen from these figures, first the robot backs up and then heads toward the virtual reference robot moving on the straight line. Figs. 5 and 6 show the simulation results for tracking a circular trajectory. The reference trajectory is a point moving counter clockwise on a circle of radius 1, starting at $q_r(0) = [x_r(0), y_r(0), \phi_r(0)]^T = [1, 0, \pi/2]^T$. The reference velocity is kept constant at $v_r(t) = 0.5$ m/s. The initial conditions for the mobile robot, however, is taken as $q(0) = [x(0), y(0), \phi(0)]^T = [0, 0, 0]^T$.

Again, as it is seen from these figures, the robot immediately heads toward the reference robot, which is moving on the circle. It then reaches it quickly and continues to track it.

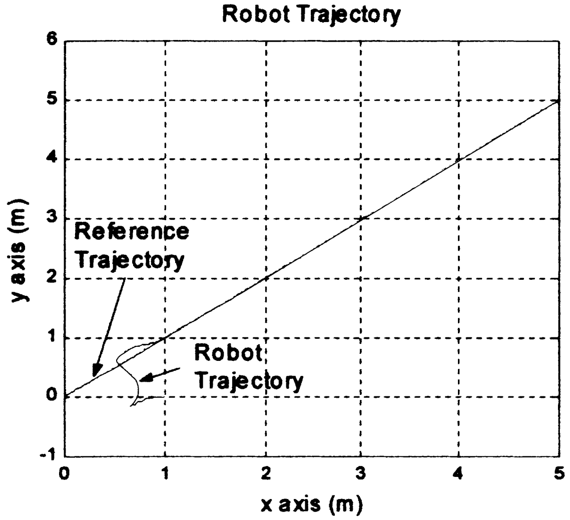

Fig. 3. Mobile robot and reference trajectories in the (x, y) plane.

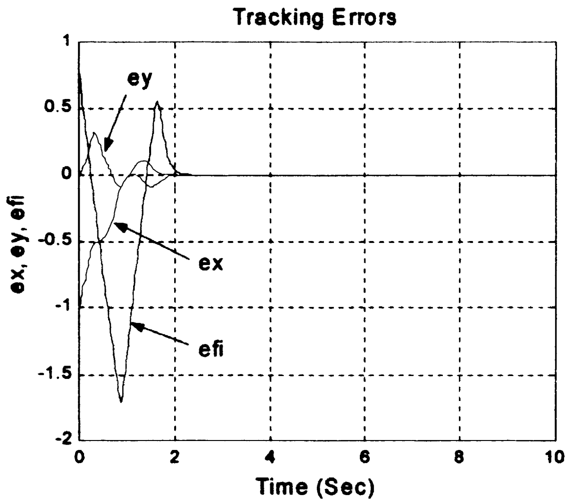

Fig. 4. Time history of the tracking errors.

6. Stabilization problem definition

The stabilization problem, given an arbitrary desired posture q_d, is to find a feedback control law, $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = u(q - q_d, \dot{q}, t)$, such that $\lim_{t\to\infty}(q(t) - q_d) = 0$, for any arbitrary initial robot posture $q(0)$. Without loss of generality, we may take $q_d = [0, 0, 0]^T$.

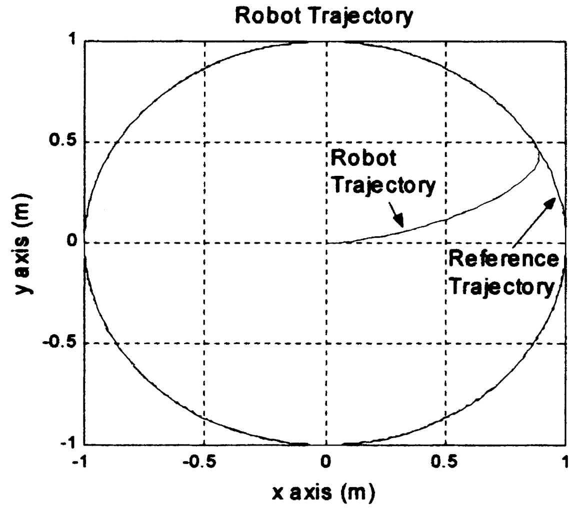

Fig. 5. Simulation results when the reference trajectory describes a circle.

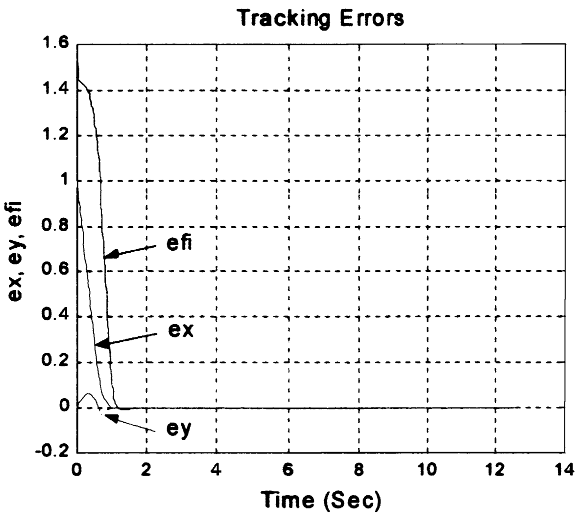

Fig. 6. Time history of the tracking errors.

6.1. Stabilization controller design

Recall that there is no continuous static state feedback that can asymptotically stabilize a nonholonomic system about a fixed posture [8–10]. The approach to the problem taken here is the dynamic extension of that in Ref. [2] where a kinematics model of the mobile robot is used. Instead of designing a new controller for the stabilization problem the same controller as for the tracking problem is used. The idea is to let the reference vehicle move along a path that passes through the point (x_d, y_d) with heading angle ϕ_d. The stabilization to a fixed posture problem is now equivalent to, and can be treated as, a tracking problem (convergence of the tracking errors to zero) with the additional requirement that the reference vehicle should itself be asymptotically stabilized about the desired posture. As in Ref. [2], we let the reference vehicle move along the x-axis, i.e. $y_r(t) = 0$ and $\phi_r(t) = 0$, for all values on t. The design method is the same as derived for the tracking case. However, in this case

$$v_r = \dot{x}_r = -k_4 x_r + g(e, t), \tag{26}$$

with

$$g(e, t) = \|e\| \sin t \tag{27}$$

where $k_4 > 0$. Different time-varying functions $g(e, t)$ have also been suggested in the literature, see Refs. [2,11] and the references therein.

Since, from the Section 5, the tracking errors e_1, e_2, and e_3 are bounded, the time-varying function $g(e, t)$ is bounded. Therefore v_r and the state x_r also remain bounded. By taking the time derivative of Eq. (26), it can be shown in the same way that $\dot{v}_r$ is bounded. Since v_r and $\dot{v}_r$ are bounded, the assumptions made in Section 3 concerning the reference velocity are fulfilled. If v_r is not equal to zero, then e must converge to zero. When e tends to zero, $g(e, t)$ also tends to zero. Therefore, the robot's position x must track x_r, which converges to zero and hence lead the mobile robot to the desired posture.

6.2. Simulation results for stabilization control problem

Here, the simulation results for the stabilization problem are shown in Figs. 7 and 8. The control parameters and system parameters are the same as for the simulations shown for the tracking problem and $k_4 = 1$. The mobile robot is initially at $q(0) = [x(0), y(0), \phi(0)]^T = [0, 1, 0]^T$.

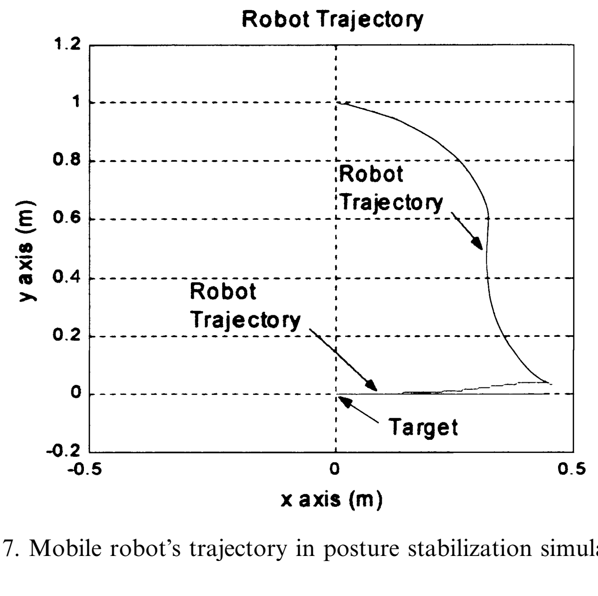

Fig. 7. Mobile robot's trajectory in posture stabilization simulation.

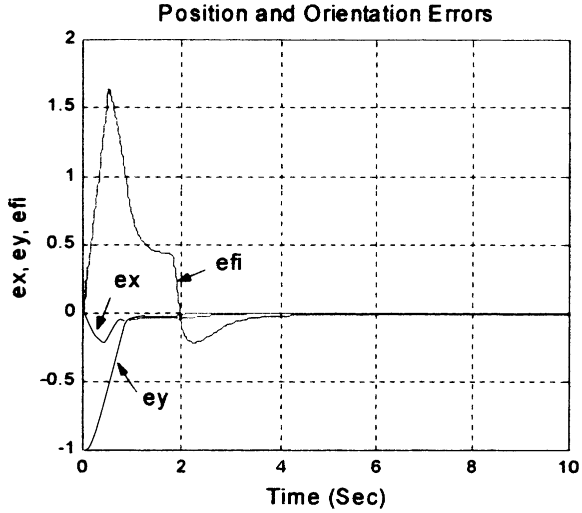

Fig. 8. Time plots of x, y, and ϕ.

As it is seen from the figures, the stabilization about the final posture at the origin is achieved quite satisfactorily. Note, in this case, that the robot actually turns around and backs up into the final posture.

7. Conclusions

Two important control problems concerning mobile robots with unknown dynamic parameters have been considered, namely, tracking of a reference trajectory and stabilization to a fixed posture. An adaptive control law has been proposed for the tracking problem and has been extended for the stabilization problem. A simple velocity/acceleration limiter was added to the controller, for practical applications, to avoid any slippage of the robot's wheels, and to improve the tracking performance. Several simulation results have been included to demonstrate the performance of the proposed adaptive control law.

References

[1] Kanayama Y, Kimura Y, Miyazaki F, Noguchi T. A stable tracking control method for an autonomous mobile robot. vol. 1. Proceedings of IEEE International Conference on Robotics and Automation, Cincinnati, Ohio, 1990, p. 384–9.

[2] Canudas de Wit C, Khennouf H, Samson C, Sordalen OJ. Nonlinear control design for mobile robots. In: Zheng YF, editor. Recent trends in Mobile robots, World Scientific, 1993. p. 121–56.

[3] Guldner J, Utkin VI. Stabilization of nonholonomic mobile robot using Lyapunov functions for navigation and sliding mode control. Control-Theory Adv Technol 1994;10(4):635–47.

[4] Colbaugh R, Barany E, Glass K. Adaptive Control of Nonholonomic Mechanical Systems. Proceedings of 35th Conference on Decision and Control, Kobe, Japan, 1996. p. 1428–34.

[5] Fierro R, Lewis FL. Control of nonholonomic mobile robot: backstepping kinematics into dynamics. J Robot Sys 1997;14(3) 149-163.
[6] Jiang ZP, Pomet JB. Combining backstepping and time-varying techniques for a new set of adaptive controllers. Proceedings of 33rd IEEE Conf on Decision and Control, Lake Buena Vista, FL, 1994. p. 2207–12.
[7] Sarkar N, Yun X, Kumar V. Control of mechanical systems with rolling constraints: application to dynamic control of mobile robots. Int J Robot Res 1994;13(1):55–69.
[8] Brockett RW. Asymptotic stability and feedback stabilization. In: Brockett RW, Millman RS, Sussmann HJ, editors. Differential Geometric Control Theory, Boston, MA: Birkhauser; 1983. p. 181–91.
[9] Bloch AM, Reyhanoglu MR, McClamroch NH. Control and stabilization of nonholonomic dynamic systems. IEEE Trans Automat Contr 1992;37(11):1746–56.
[10] Campion G, d'Andrea-Novel B, Bastin G. Controllability and state feedback stabilization of nonholonomic mechanical systems. Canudas de Wit C, editor. Advanced Robot Control, Berlin: Springer; 1991. p. 106–24.
[11] Kolmanovsky I, McClamroch NH. Developments in nonholonomic control problems. IEEE Contr Sys Magaz 1995;15(6):20–36.
[12] Krstic M, Kanellakopoulos I, Kokotovic P. Nonlinear and Adaptive Control Design, New York: Wiley; 1995.
[13] Karlsson MP. Control of nonholonomic systems with applications to mobile robots. Master Thesis, Southern Illinois University, Carbondale, IL 62901, USA, 1997.

Farzad Pourboghrat received his Ph.D. degree in Electrical Engineering from the University of Iowa in 1984. He is now with the Department of Electrical and Computer Engineering at Southern Illinois University at Carbondale (SIU-C) where he is an Associate Professor. His research interests are in adaptive and sliding control with applications to DSP embedded systems, mechatronics, flexible structures and MEMS.

Mattias Karlsson received the B.S.E.E. degree from the University of Borås, Sweden and the M.S.E.E. degree from Southern Illinois University, Carbondale, IL, in 1995 and 1997, respectively. He has been employed at Orian Technology since 1997. He is currently an on-site consultant at Caterpillar Inc.'s Technical Center, Mossville, IL. His current interests include control algorithm development for mechanical and electrical systems and software development for embedded systems.